Quality Assurance in Measurement

Module 3 - Part II - Statistical Methods in Measurement - Uncertainties

Contents

13.0 Uncertainty of Measurement

It is a fact that every measurement has some uncertainty associated with it – how much depends on the measuring instrument used as well as how it is used. Calculating the uncertainty of a measurement is important for being assured of:

1. good quality measurements,
2. whether a specification is likely to be met.

In the next section, we explore more fully what can influence the uncertainty of a measurement and see how to construct an **uncertainty budget** i.e. a list of all possible contributors to the overall uncertainty of a measurement. For now we concentrate on how an uncertainty is calculated.

Estimates of the uncertainty are obtained in one of two ways:

1. from repeated readings – **Type A –** giving what might termed "***repeatability uncertainty***"
2. using information from past experience, calibration certificates, manufacturers' specifications, calculations derived from manufacturers' information – **Type B**

These individual estimates of uncertainty can combined to produce an overall uncertainty of measurement. We have already established that we can estimate the true value of a measurement by the sample mean $\bar{x}$ of n observations. The uncertainty of this estimate is expressed in terms of a "margin of doubt" – i.e. a confidence interval with its level of confidence (as a percentage) to describe the confidence the width of the uncertainty confidence level encloses the true value. The confidence level is usually about 95% or about 99.7% ("about" because the measurement estimate only approximately follows the normal distribution) giving a confidence interval of:

a. $\pm\ 2s/\sqrt{n}$ for a 95% uncertainty confidence level
b. $\pm\ 3s/\sqrt{n}$ for a 99.7% uncertainty confidence level

Confidence intervals for the uncertainty of a measurement are sometimes referred to as ***expanded uncertainty***.

(Such expressions are familiar to us from our discussion on the standard error on the means!)

The factors "2" and "3" are commonly known (in measurement circles!) as a coverage factor, k. Whether you choose k = 2 or k = 3 depends upon the level of uncertainty you are prepared to accept – either way, the coverage factor must always be stated. By the way, a coverage factor of 1 (k = 1) would correspond to about a 68% level of confidence.

So for example, a CMM with an expanded uncertainty (k = 2) of ± 0.0002mm measures a length as 121mm and so a 95% uncertainty confidence level is 121 ± 0.0002mm.

13.1 Combining Uncertainties

In this section we see how to obtain a measure of "overall uncertainty" in a measurement that is subject to more than one source of error. We will first discuss the situation where these component errors are combined linearly before summarizing how more involved combinations are treated.

First let us suppose we have two independent random variables X and Y with means μ_x and μ_y and variances var(X) and var(Y). Then it can be shown that

(a) for the sum X + Y, the mean is given by $\mu_x + \mu_y$ and the variance of X + Y is var(X) +var(Y);

(b) for the difference X – Y, the mean is given by $\mu_x - \mu_y$ and the variance of X - Y is var(X) + var(Y).

We will illustrate the second of these results for two simple random variables X and Y given by:

X is equally likely to take the values 4 5 5 6.

Y is equally likely to take the values 1 2 3

Now $\mu_x = 5$ and var(X) = ½ while $\mu_y = 2$ and var(Y) = ⅔.

The possible (equally likely) values of the random variable X - Y are:

1 2 2 2 3 3 3 3 4 4 4 5.

Activity 13.1 *Calculate the mean and the variance of X - Y and hence verify the statement in (b) above.*

How can these results be used in a practical situation to obtain an overall measure of uncertainty? Well, suppose we have three independent sources of error in a measurement that combine additively to form the "final" error. Then if these errors are represented by the random variables X, Y and Z the total variance is given by var(X) + var(Y) + var(Z). Thus if we have estimates for the three standard errors, se_x, se_y and se_z a measure of the overall uncertainty is given by

$$3\sqrt{se_x^2 + se_y^2 + se_z^2}$$

Note that for calibration errors we would use a theoretical standard deviation in place of a standard error. For example, suppose that in a particular measurement two sources of error have been identified, an elementary systematic error and a measurement error. The elementary systematic error is quoted as 0.2 and thus the standard deviation of the resulting error is given by 0.2/√3 (using the result for the standard deviation of a uniform distribution over (-0.2, 0.2)), while the standard error of the measurement error has been estimated by experiment as 0.16. Using the result above we can estimate the overall uncertainty as being

$$3\sqrt{\frac{0.2^2}{\sqrt{3}} + 0.16^2} = 0.77$$

Of course, the above only holds if the component errors (the random variables, in the statistical jargon) are independent. If any of the component sources of error are related then naturally we cannot use the above formula since it is based on independence.

Suppose we have two random variables i.e. sources of error, X and Y that are believed to be dependent. Then it can be shown that

$$\text{var}(X \pm Y) = \text{var}(X) + \text{var}(Y) \pm 2\text{cov}(X, Y).$$

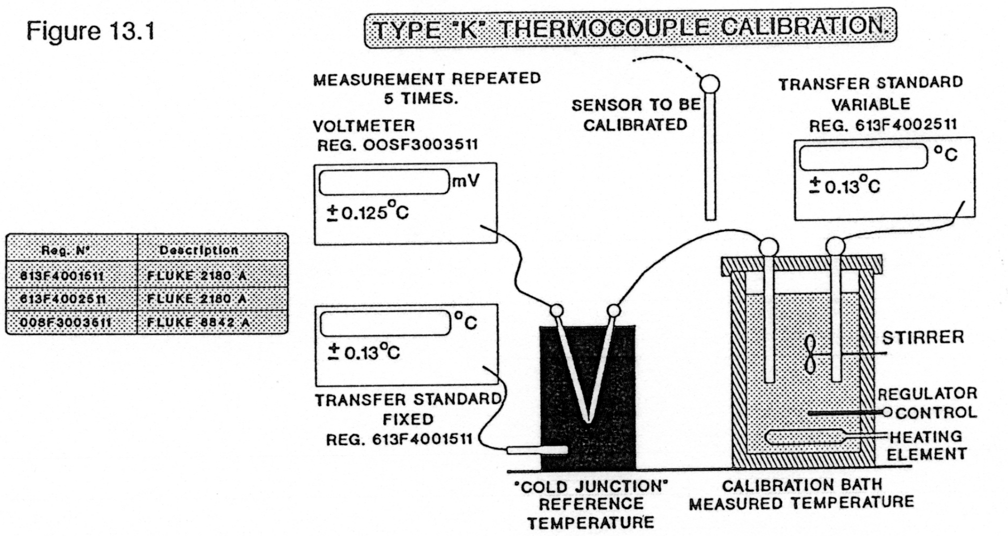
The cov(X,Y) is estimated from the observed measurements using the formula

$$\text{cov}(X, Y) = \frac{\sum(X - \bar{X})(Y - \bar{Y})}{n - 1}$$

13.2 Case Study: Calibration of a Thermocouple

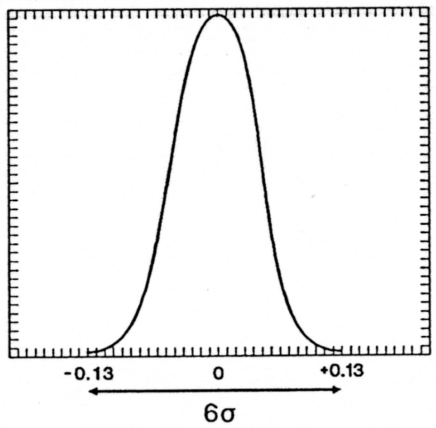
The situation is summarised in Figure 13.1.

Figure 13.1



Here we have identified four sources of error. The probable sizes of three of these are quoted by the manufacturer, Fluke. However, in this case a quoted error of 0.13°C (say) does not relate to an elementary systematic error but to the "effective range" of a normal distribution as illustrated in Figure 13.2.

Figure 13.2



From this we derive an estimate for σ of 0.13/3.

The four sources of error are:

1. The Standard (fixed) with a quoted error of ± 0.13°C giving an estimated standard deviation of s_1 = 0.13/3.
2. The Standard (variable) with a quoted error of 0.13°C giving an estimated standard deviation of s_2 = 0.13/3.
1. The voltmeter which has a quoted error of 0.13°C giving a standard deviation of s_3 = 0.13/3.
2. The repeatability error (se_5) that is estimated from five readings.

Thus our overall uncertainty of a measurement is given by:

$$h_e = 3\sqrt{\{s_1^2 + s_2^2 + s_3^2 + s_4^2 + s_5^2\}}$$

The table below gives the results of the experiment.

standard (variable)	standard (fixed)		Voltmeter readings		repeatability error	uncertainty	
Xo	Sf	Xi	x	x	se5	se5	he
°C	°C	mV	mV	°C	mVx10^{-4}	°C	°C
60.11	22.42	1.555					
60.11	22.42	1.554					
60.11	22.42	1.554	1.554	60.33	7.071	0.020	
60.11	22.42	1.554					
60.11	22.42	1.553					
99.75	21.95	3.240					
99.75	21.95	3.239					
99.75	21.95	3.239	3.2396	100.51	5.477	0.013	
99.75	21.95	3.240					
99.75	21.95	3.240					
124.37	22.30	4.246					
124.37	22.30	4.247					
124.37	22.30	4.246	4.246	125.32	7.071	0.020	
124.37	22.30	4.245					
124.37	22.30	4.246					
151.19	22.62	5.321					
151.18	22.62	5.322					
151.19	22.61	5.322	5.322	152.19	7.071	0.020	
151.19	22.62	5.323					
151.19	22.62	5.322					
181.74	21.77	6.571					
181.73	21.77	6.572					
181.74	21.77	6.571	6.5714	182.57	5.477	0.013	
181.74	21.77	6.572					
181.74	21.77	6.571					
196.64	22.36	7.141					
196.64	22.36	7.142					
196.64	22.36	7.141	7.141	197.43	7.071	0.020	
196.64	22.36	7.140					
196.64	22.36	7.141					

Activity 13.2 *Verify that the uncertainty of measurement for the first row of the above table is 0.2303. Calculate the uncertainties for the remaining rows. Comment on the results.*

13.3 Estimating the Standard Error of a Compound Quantity

We conclude this module with the a table giving (approximate) standard errors for quantities which consist of products and other functions of two or more measurable quantities.

Suppose we have independent measurable quantities A, B, C etc whose means are m_a, m_b, m_c etc. and whose standard errors are α_a, α_b, α_c etc. respectively. Then the standard error of:

(a) the sum A +B is $\sqrt{\{\alpha_a^2 + \alpha_b^2\}}$

(b) the difference A - B is $\sqrt{\{\alpha_a^2 + \alpha_b^2\}}$

(c) the product AB is $\sqrt{\{m_b^2\alpha_a^2 + m_a^2\alpha_b^2\}}$

(d) the product ABC is $\sqrt{\{m_b^2 m_c^2\alpha_a^2 + m_a^2 m_c^2\alpha_b^2 + m_a^2 m_b^2\alpha_c^2\}}$

(e) the quotient A/B is $\dfrac{\sqrt{\{\alpha_a^2 + m_a^2\alpha_b^2\}}}{m_b}$

(f) the power A^p is $[m_a^{\,p-1}]\alpha_a$

(g) any function of A, B, ... , Z denoted by f(A, B, ... , Z) is given by α where

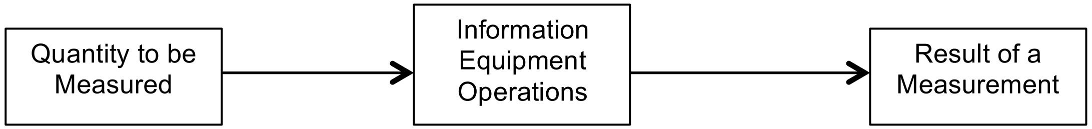
$$\alpha^2 = \frac{\partial f}{\partial A}^2 \alpha_a{}^2 + \frac{\partial f}{\partial B}^2 \alpha_b{}^2 + \cdots ..$$

Activity 13.3 A number of measurements of the radius and length of a cylinder have yielded a mean radius of 2.1 cm with a standard error of 0.1 cm and a mean length of 6.4cm with a standard error of 0.2cm. Show that the mean volume of the cylinder is approximately 88.7cm^3 with a standard error of approximately 8.9cm^3. What is the overall uncertainty in the measurement of the volume of the cylinder?

14.0 The Measurement Process

The measurement process is defined in the International Vocabulary of Basic and General Terms in Metrology as all the information, equipment and operations relevant to a given measurement (figure 2.1)

Figure 14.1



In planning for quality of measurement it is necessary to establish the acceptability and capability of the measurement process by calculating the uncertainty of measurement. And to do so it is necessary to use methods that identify and take into account the various elementary errors that arise in the process. We are therefore looking to identify, quantify and characterise the following;

- elementary errors of the measuring instrument, chain or system (ICS), as appropriate, under fixed reference conditions,
- elementary errors of the measuring instrument, chain or system (ICS), as appropriate, under variable or ambient conditions (e.g. on site)
- other elementary errors of the measurement process arising from the definition of the measurand, the method of measurement, the operating procedure, the operators, the location and the conditions of use.

All of the elementary errors, contributing to the uncertainty of a measurement result, may be conveniently summarised in the form of a cause and effect diagram, as shown in figure 14.2.

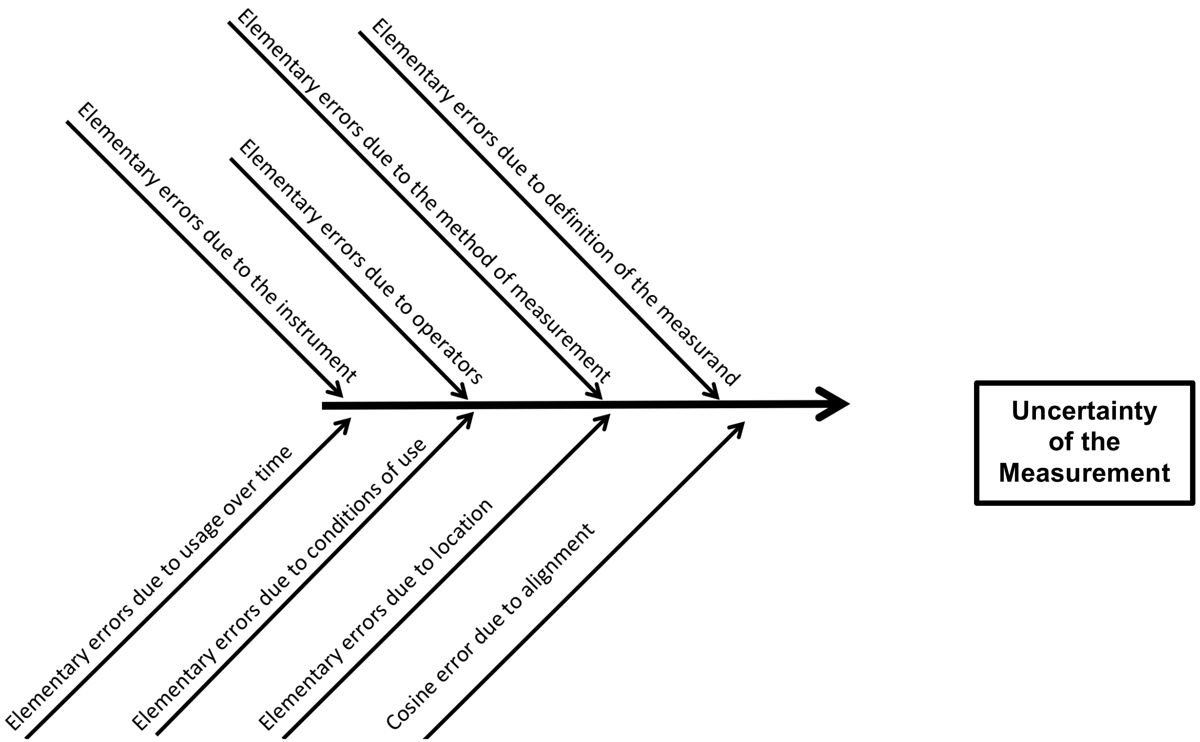


Figure 14.2

Although reference is made to instrument errors in figure 14.2, it could equally have been made with respect to a measuring chain or system, depending upon what is actually being used for a particular measurement. Elsewhere in the module where this applies the simple expedient of using ICS as an abbreviation for measuring instrument, chain or system will be applied.

14.1 Repeatability and Reproducibility of the Measurement Process

Repeatability and reproducibility are terms that are often confused. It Is important to distinguish between the two and appreciate their role in the scheme of things. Both are concerned with the closeness in agreement between the results of successive measurements on the same measurand, but differ in the conditions under which the measurements are made.

The **REPEATABILITY** of measurement is defined (PD6461) as **the closeness of agreement between the results of successive measurements of the same measurand carried out subject to all of the following conditions;**

- the same method of measurement
- the same operator
- the same measuring standard, instrument, chain or system
- the same location
- the same conditions of use
- repetition over a short period of time

REPRODUCIBILITY on the other hand is defined (PD6461) as **the closeness of agreement between results of measurements of the same measurand, where the individual measurements are carried out changing conditions such as**:

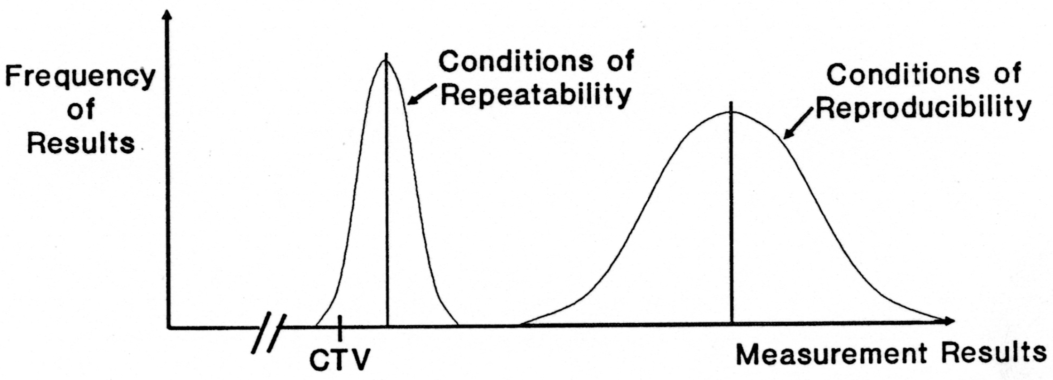
- the method of measurement
- the operator
- the measuring instrument
- the location
- the conditions of use
- the time

Note: *For the statement of reproducibility to be valid it is necessary to specify the conditions that are changed.*

Since the conditions of reproducibility are more closely aligned to the circumstances encountered when a measurement is taken "on site", you may wonder why we should concern ourselves with the assessment of repeatability. The simple answer is that the repeatability provides us with an indication of the best we can get, for a given measurement process, without the additional variability introduced by changes in "outside" influences. If the repeatability results indicate that the measurement process is inadequate for the task to be performed, then it is inappropriate to proceed to applying it or even to the stage of evaluation under conditions of reproducibility. What is indicated therefore in such situations is a need for a new or improved process.

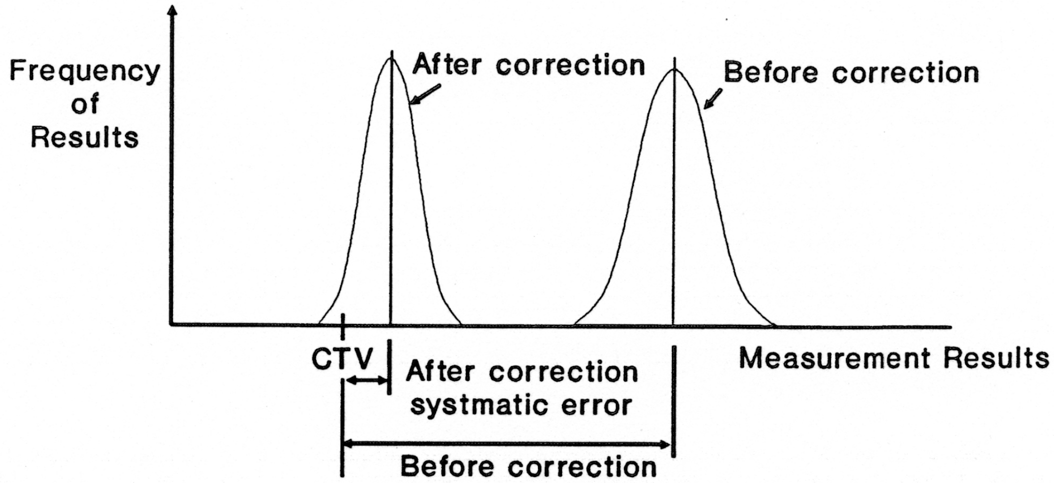
Under conditions of reproducibility information is obtained on the influence of changeable conditions upon the measurement results. In general the effect is an increase in dispersion over the results obtained under conditions of repeatability and, quite often, the influence of additional systematic components of error (figure 14.3).

Figure 14.3



Equipped with this information appropriate actions may be taken to correct for systematic errors and/or seek to reduce dispersion if this is considered excessive in relation to the on-site measurement requirements and the tolerance limits specified for the measurand (figure 14.4).

Figure 14.4



In view of the importance of repeatability and reproducibility it is appropriate to identify the quantitative data that can be determined from these operations for the measurement process.

14.1.1 Repeatability of the Measurement Process

When dealing with the measurement process the procedure requires several measurements to be taken upon the same conventional true value (CTV), conventional reference value (CRV) or reference value (RV). From the results obtained it is possible to determine; + the random error of repeatability of the measurement process (3sr),

- the systematic error of repeatability of the measurement process, $d_r = \bar{x}$ - CTV, and
- the uncertainty of the measurement result for the measurement process, under the conditions of repeatability, $|d_r| + 3s_r$

14.1.2 Reproducibility of the Measurement Process

By performing the measurements on the same CTV, CRV or RV under conditions of on-site variability, the data obtained can be used to determine;

- the random error of reproducibility of the measurement process, s_g,
- the systematic error of reproducibility of the measurement process, d_g, and
- the uncertainty of the measurement result for the measurement process, under conditions of reproducibility, hg = $|d_g| + 3s_g$

15.0 The Instrument, Chain or System (ICS)

Instruments, or in more general terms the Instrument, chain or system (ICS), are integral components of the measurement process. As Indicated earlier in the cause and effect diagram of figure 2.2 the ICS is a source of elementary errors that contribute to the overall uncertainty of the measurement process. The ICS may be operated under various conditions and it is important to specify these conditions and be aware of the implications that they impose. For specifying the utilisation of an ICS the following categories of conditions should be considered;

- rate operated conditions
- limiting operating conditions
- reference conditions.

The international vocabulary of general fundamental metrology terms provides the following definitions for these conditions.

- **Rate operated conditions:** Operating conditions giving ranges of quantities to be measured, ambient conditions and other important requirements for which metrological characteristics of a measuring instrument (ICS) are supposedly maintained within specific limits.
- **Limiting operating conditions:** Extreme conditions that a measuring instrument (ICS) can withstand without damage and without deterioration of its metrological characteristics when it is subsequently used under conditions prescribed for its operation.
- **Reference conditions:** Conditions of utilisation for a measuring instrument (ICS) prescribed for operational tests or to validly ensure comparisons of measurement results between them. The reference conditions are the conditions under which the characteristics of an ICS are generally given, but can vary from one manufacturer to another.

In considering the conditions under which an ICS operates, it is important to identify, and consider the influence of, the surrounding or ambient conditions, such as temperature, atmospheric pressure, relative humidity, vibrations and shocks, dust and electromagnetic conditions. The International Electrochemical Commission (CEI) has produced a classification scheme for ambient conditions and recommendations for their use. These are covered more fully in section 16.0.

The errors associated with the instrument (or ICS) under various conditions are evaluated during a calibration procedure. Unfortunately, the calibration itself introduces errors that lead to uncertainty in the evaluation of the systematic and random errors of the instrument. To minimise this uncertainty it is therefore necessary to reduce the contribution of errors during calibration. Two sets of conditions are generally defined and used for calibration, which parallel in many respects the conditions Identified for repeatability and reproducibility of the measurement process.

15.1 Evaluation of the ICS under reference conditions

Because of its central role in deriving a measurement result it is important to know what errors are likely to be introduced by the ICS and how they compare with other contributory errors within the measurement process. By calibrating the Instrument (or ICS) under both reference conditions and variable conditions this particular Information can be obtained.

Evaluation under the reference conditions requires:

- the same operator,
- the same method of measurement,
- the same location, the same conditions of use, and
- repeated measurement over a short period of time.

Repeated measurements are performed upon the same CTV or CRV and from the data obtained the following information can be derived:

- the systematic error of the ICS, under the reference conditions, d_f
- the random error of the instrument, under the reference conditions, e_f = 3sf, and
- the uncertainty of the ICS, under reference conditions; $h_f = |d_f| + 3s_f$

15.2 Evaluation of the ICS under variable conditions

In these circumstances we are concerned with evaluating the ICS under variable conditions such as those encountered on-site, but maintaining:

- same operator,
- same method, and
- same location

By repeating the calibration of an ICS under the variable, but defined, conditions the following information can be obtained:

- the systematic error of the ICS, under the variable conditions, d_v

- the random error of the ICS, under conditions of reproducibility, $e_v = 3s_v$, and
- the uncertainty for the ICS, under conditions of reproducibility; $h_v = |d_v| + 3s_v$

Having set the scene by drawing attention to the information that can be gained from considering the ICS under reference and variable conditions lets now consider the ways in which the elementary errors can be identified, quantified and subsequently characterised for the ICS.

15.3 Identification of Elementary Errors of the ICS under reference and variable conditions

Having established what constitutes the uncertainty of a measurement result under reference and variable conditions it is important to consider further the way in which we identify the elementary errors that are likely to be critical in the measurement we are planning. The cause and effect diagram presented in section 2.0 (figure 2.2) provides a useful guide to what we should be looking for, but we need to go deeper, by looking more closely at the contributory branches. We need to identify all the errors Inherent in the measuring instruments to be used within the measurement process, that are likely to have a noticeable influence upon the overall uncertainty of the measurement result. These errors may be static or dynamic errors, including, for example:

- non-linearity errors
- hysteresis errors
- errors in resolution
- zero or bias errors
- response time errors, and so on.

The information we need to identify these errors is obtained through evaluation and an important source of such Information is the evaluation report for the measuring element or elements concerned. If appropriately produced this will include detailed reference to errors and their representative values.

In the event that an evaluation report is not available it is then necessary to refer to manufacturer test results, if available, or possibly to the technical documentation for the element or elements concerned. If it is not possible to obtain the Information form these sources it may then be necessary to undertake tests or even a full evaluation of the element concerned using procedures identical to those used in calibration.

Once we have identified the elementary errors within our measurement process we then need to assign values and express them in a way that is both meaningful and useful. Two particular ways may be identified for doing this:

- by expressing the maximum error values in the form $\pm e_{max}$, or

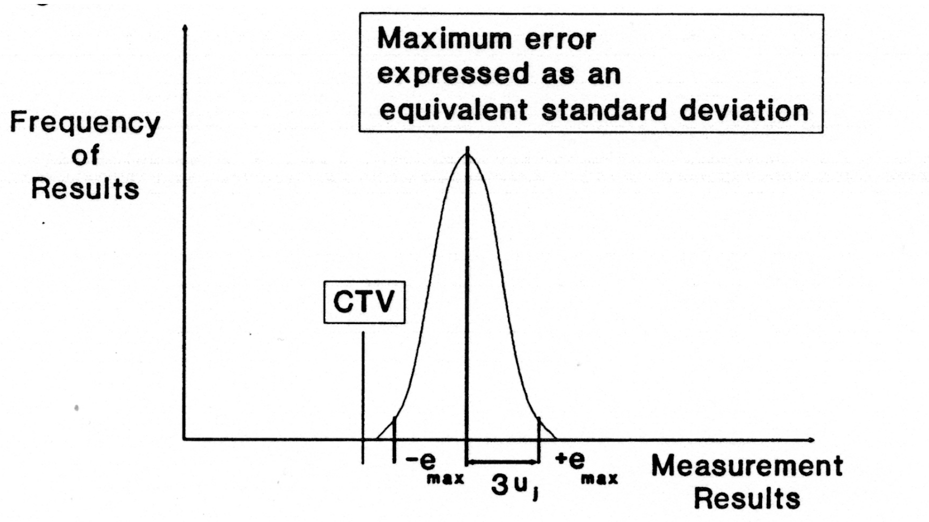
- by estimating, from a series of measurements, the standard deviation, s_r,

In view of the need to determine the overall uncertainty of the measurement process it is quite usual when presented with only maximum error data to express it in the form of what essentially amounts to an "equivalent standard deviation", u_j, by using the relationship:

$$\frac{u_j}{3} = e_{max}$$

The justification for this is open to question, but in practice it has been shown to be a useful convention. The assumption on which this relationship is based Is simply that the maximum error may be considered to correspond with the difference between the mean and the tail value, three standard deviations from the mean, in a normal distribution (figure 15.1).

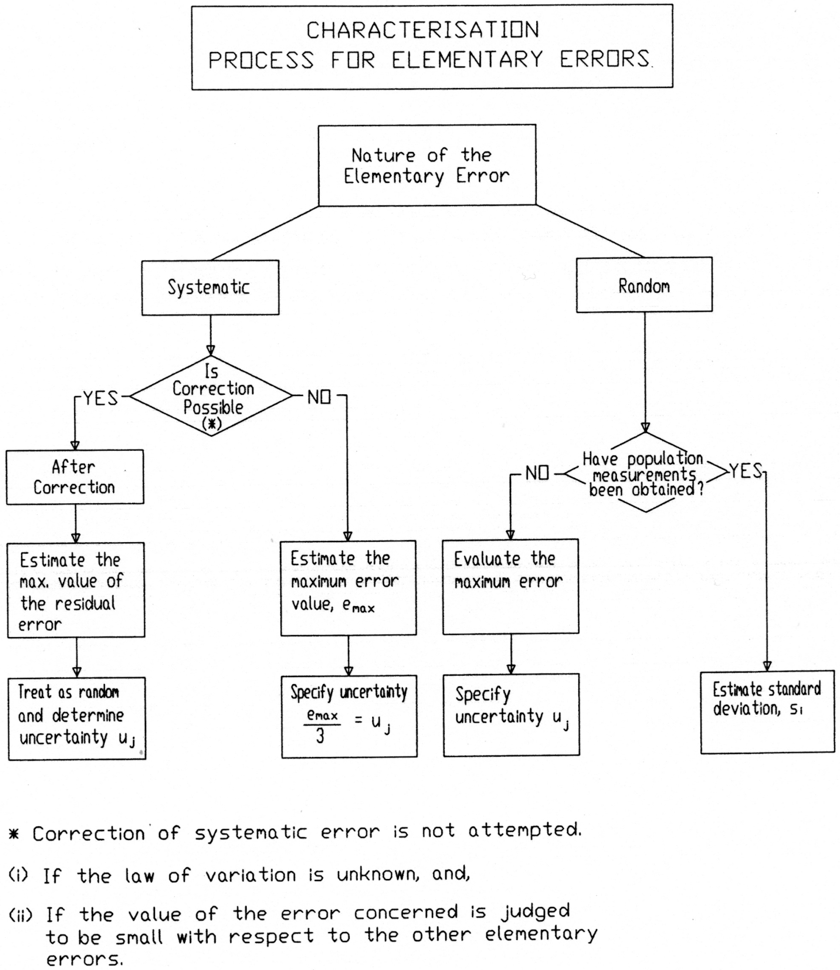
Figure 15.1



To complete the picture concerning elementary error terms of the ICS under reference conditions it is necessary to characterise them; to specify them as either systematic or random in nature.

15.4 Characterisation of elementary errors of an ICS under reference conditions

Having identified the elementary errors of an ICS the next stage in the planning procedure Is to characterise these errors, under conditions of reference. In this way we are characterising the Instrument, chain or system unobscured by the influences of outside variable effects. To do this we need to determine whether an Identified error is random or systematic in character and ascribed to it a standard deviation, s_i or an "equivalent standard deviation", u_j. The diagram shown below summarises the procedure for characterisation.



One or two simple examples will illustrate the point.

Linearity error is an example of a systematic error, which can vary over the range of the measurement (figure 15.2).

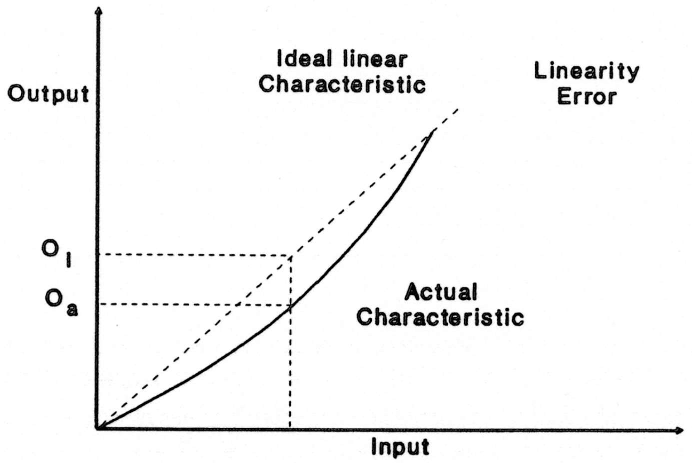


Figure 15.2

The non-linearity in this case may be expressed as the difference between the ideal output for a given input and the actual output, for a particular point within the range,

$$N\ (I) = O_a - O_i$$

where O_a is the actual output and O_i the output for the ideal straight line relationship.

It can be seen from this that, for a chosen point within the range, the error is fixed and accountable and it is therefore reasonable to consider it as a systematic error. In treating it as such it is appropriate to compensate for the effect, in this case requiring a correction table for the full range of the measurement or a functional algorithm for automatic correction. Alternatively, it may be acceptable to take account of the errors by quoting or applying, in uncertainty terms, the maximum value of error.

Where a residual error is reduced to a small value by correction it is generally treated as a random component in determining the overall uncertainty of a measurement result.

If the systematic error cannot be corrected it is usual to express the maximum value of the error in terms of an equivalent standard deviation $e_{max} = 3u_j$

In dealing with random errors within the ICS it is appropriate, where possible, to estimate the standard deviation, s_i, and so characterise the error in this way. However, there may be occasions in which the error may be considered to be random, but in the absence of population data cannot be characterised by means of a standard deviation. In these cases it may be appropriate to evaluate a maximum error and express it as an "equivalent standard deviation", u_j.

It is important to note in expressing either s_i or e_j components that they must be expressed in quantities and units that are consistent for the purpose of determining overall uncertainty of a measurement result.

15.5.1 Accuracy and Resolution

The reading showing on a car speedometer can be quite dubious! Most of the analogue speedometers look something like the image on the left:

Over the years, we have that the accuracy of such speedometers varies according to the manufacturer. For instance, a Rolls Royce Silver Shadow the true speed is 10% lower than the indicated; with a Skoda Fabia the error is 2% while with a Vauxhall Antara 5%. In other words, all these speedometers have non-linear accuracy which if known a correction to the observed speed can be applied to estimate the true speed – this is quite useful in a 30mph zone where the police often apply a “zero tolerance”!

Is a digital speedometer more accurate? There is an illusion that it is! – a digital number seems better than dial. Well, no! The digital speedometer in the Nissan Leaf (electric car) shown above on the right has error of about 11%

Also, digital speedometers have **resolution** limitations. For example, while an analogue speedometer can theoretically display an infinite number of speeds between say, 30 and 40mph, digital speedometers generally display speeds in whole numbers.

Basically, this means that with an analogue speedometer displaying about 28mph the uncertainty of measurement would be estimated from a normal distribution having a range of, say, 2mph (estimated while driving!). This would give a resolution uncertainty (error) of 1/3 mph.

On the other hand, with a digital speedometer displaying 28mph the actual recoded speed could be equally likely to lie between 27.5 and 28.4999. Thus, to estimate standard uncertainty (error) we would use the result for the uniform distribution with a range (-a, a) that the standard deviation is a/√ 3. So with the speedometer having an error range of 1 the resolution uncertainty is 0.5/√ 3.

3.5 Documentation of elementary errors of an ICS identified under set reference conditions

Documentation is an essential part of the Quality Assurance process and the importance of good documentation cannot be over emphasised. It is important in planning, implementation, control and overall management of the measurement process.

Characterisation of elementary errors for the measurement process can be conveniently summarised in a table below. Such a document would form part of a full and comprehensive Measurement Process Documentary Structure (MPDS), which is identified as an ensemble of dossiers used as a control document for the measurement process.

The form illustrated In figure 15.4 is suitable for recording identified elementary errors of reference quantities and the other elementary errors encountered in a measuring process, as well as errors of the ICS or the elements of a measuring chain. In the particular example shown the facility is provided for specifying whether the error is random or systematic, and how the systematic component is treated if corrected or uncorrected. The table also has columns for entering the appropriate values of u_j or s_i.

In considering the entries for s_i and u_j the procedure is as follows:

- For random errors, where distribution data is unavailable, the maximum error is entered in the "random" column and the same value divided by 3 is entered into the u_j column.

- For random errors, where distribution data is available, the estimator value (standard deviation) is entered in the random column and again within the s_i column.

- For systematic errors that are uncorrected, the value of the maximum error is entered into the "uncorrected column" and an equivalent standard deviation value of one-third the maximum error is entered in the u_j column.

- For systematic errors that are corrected, the value following correction Is entered Into the "corrected" column and an equivalent standard deviation value, of one third the corrected value, is entered into the u_j column.

The implication arising from the treatment of systematic maximum errors as equivalent standard deviations is that they are being viewed as random components. However, care has to be exercised in handling these components in deriving values for uncertainty. The distinction on the basis of being either s_i or u_j allows us to do this and in combining them to obtain a value of uncertainty the process of quadrature addition is applied. Thus, for a series of si components to be combined with a series of uj components, to obtain an uncertainty value, h_r, the expression used is:

$$h_r = 3\sqrt{\sum s_i^2 + \sum u_j^2}$$

SUMMARY TABLE OF ELEMENTARY ERRORS.

Reference of the measurement process under study:
Date:
Name:

Elementary Error taken into consideration	Random	Systematic		u_j	s_i
		Uncorrected considered as random	Residual after Adjustment (1)		
Error on the reference					
Errors due to measuring instruments - sensitivity - linearity - resolution . . . . - temperature on the zero - temperature on sensitivity - time					
- Errors due to the measurement process - Procedure - Operators					

(1) Specify the value of the correction made.

<u>Note:</u> u_j and s_i are estimators of the component standard deviations of the standard deviation refering to <u>uncertainty</u> of measurement.

16.0 Uncertainties of Measurement of the Measurement Process

So far the identification, quantification and characterisation procedure has been considered for the ICS under reference and variable conditions. Full consideration of the measurement process, under the conditions of reproducibility, requires broader attention to influential factors, by the planners of the measurement process in association with the person or persons who specified the need for the measurement.

Within the range of considerations to be applied to the planning of the measurement process and the identification of elementary errors will be the following:

- the measurand and how it may be Influenced by variable conditions, and
- the most influential variable conditions to be included in the assessment of uncertainty.

16.1 Elementary Errors and Uncertainty of the measurement process

In determining the elementary errors of the measurement process various tests may have to be applied to estimate the values and their respective influence upon the measurement process. These errors, due to the variable conditions of reproducibility, must be characterised as random or systematic components and expressed in the form of estimators of their respective standard deviations, s_i and u_j,. as described in section 3.4, for the ICS.

Using the table, described in section 15.5, a complete listing of the s_i and u_j values can be assembled, and so provide the following information:

- errors associated with a particular standard, certified reference or reference value, and otherwise known as the errors of reference,
- errors of the ICS under variable ambient conditions, and
- the other errors of the measurement process due to the variable conditions of reproducibility.

To complete the table it is also appropriate to specify corrections that can be applied to Identified systematic errors.

Once the "error table" or *uncertainty budget* has been completed, the uncertainty of the measurement process can be calculated, and expressed in the form,

$$h_g = |d_g| + 3s_g$$

where d_g is the systematic error in the process after corrections, where possible, have been applied.

The result of a measurement may then be expressed as

$$\text{result} = \text{corrected result} + \text{uncertainty } (h_g)$$

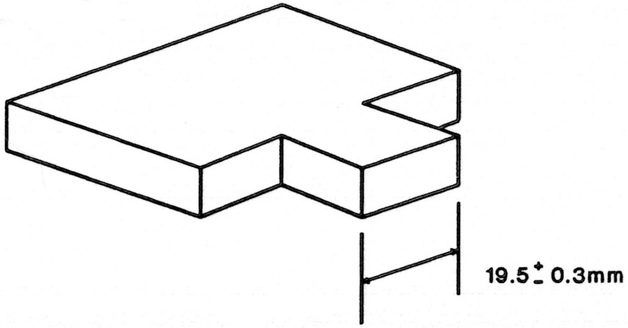
16.2 Case Study - Measurement of a Tenon component using a Vernier Caliper

The following case study provides a description of the procedures for identification and calculation of the following;

- Random error
- Systematic error
- Standard deviation
- Uncertainty of measurement

The study is based upon the measurements obtained from an aluminum tenon component, illustrated in Figure C1.

Figure C1



Measurements were made using a vernier caliper having the following characteristics;

- Measurement range: 0 - 400mm
- Resolution: 0.02mm
- Errors (in reference conditions: 20°C ± 0.5°C, 55% humidity ±5%): ± (20 +0.02L) x10^{-3}mm where L = length measured
- Errors (in variable temperature conditions): 11.5μm $°C^{-1}$ m^{-1} for steel)

Considering further the errors inherent in the vernier, the ISO Standard for Vernier Callipers (ISO 6906) states that for a vernier caliper capable of reading to 0.02mm the maximum error in reading may be obtained from the expression:

$$e = \pm (20 + 0.05L) \times 10^{-3} \mu m$$

where L is the length measured, which in this case is 19.5 mm.

Hence the error, e = ± (20 + 0.05 x 19.5) μm
= ± 20.975μm

The measurements on the component were taken at 27°C, 7°C above the reference temperature, making It necessary to consider the influence of the expanded vernier on the measurement result. Taking the temperature coefficient of linear expansion for the steel caliper as 11.5 mm°C^{-1} m^{-1}, the expansion value is,

$$(11.5 \times 7) \times 19.5 \times 10^{-3} \mu m = 1.57 \mu m$$

The vernier error and the associated error due to temperature together constitute the instrumentation errors involved in the measurement.

Turning now to the process errors, consideration has to be given to:

- the influence of temperature on the component dimensions
- error in the definition of the component dimensions
- error in the method of measurement, and
- the operator error

The temperature coefficient of expansion for aluminum is roughly twice that of steel, being approximately 23μm °C^{-1} m^{-1}. Thus for a 7°C rise above the reference conditions the aluminum expands,

$$23 \times 7 \times 19.5 \times 10^{-3} \mu m = 3.14 \mu m$$

In considering the implications of expansion for both the vernier and the component it can be seen that the overall error from these two sources is 1.57μm. This is because the vernier expanded by 1.57μm and the aluminum by twice this value, so that half of the error associated with the component expansion is, in this case, cancelled out by the expansion in the vernier.

In addition to the Influence of temperature on the aluminum component it was also established that an uncertainty In the definition of the size of the component could be ascribed, adding an extra 2.5μm to the possible error. This, together with an identified error In the measuring process of 10μm, is presented in the table below.

	Error Description	**Error Value**	**u**	**s = u/3**
Instrument Errors	Vernier Error (ISO 6906) Expansion Error	20.975 µm 1.57 µm	20.975 µm 1.57 µm	6.992 µm 0.52 µm
Process Errors	Expansion Error Error in definition of component dimension Error in method of measurement Operator Error	3.14 µm 2.5 µm 10 µm 10 µm	 2.5 µm 10 µm 10 µm	 0.83 µm 3.33 µm 3.33 µm

In looking at the combination of "error components" to determine uncertainty for the measurement it Is necessary, first of all, to determine which are systematic errors and which are random. All but the operator errors are In this case considered random, and treated accordingly. Thus the standard deviation for the process is:

$$s = \sqrt{[48.88 + 0.27 + 0.69 + 11.09]}\ \mu m$$
$$= \sqrt{60.93}\ \mu m$$
$$= 7.61\ \mu m$$

The uncertainty due to these errors, expressed at the 99.73% confidence level is

$$3s \quad = 23.42\ \mu m$$

But this excludes the influence of the systematic error due to the operator. To include this it is simply necessary to add it algebraically to the uncertainty value estimated above. Hence, the total uncertainty of the measure is,

$$h_g = (10 + 23.42)\ \mu m$$
$$= 33.42\ \mu m$$

Should it be considered appropriate to treat the operator error as a random component, and in the absence of any other systematic error, the uncertainty is

$s = 3 \sqrt{[48.88 + 0.27 + 0.69 + 11.09 + 11.09]}$
$= 25.47 \mu m$

16.3 Uncertainty Calculations for Direct Methods of Measurement

The method of measurement has an important and direct bearing upon the uncertainty of the measurement process. Where a single measurand is involved and the value is obtained directly, without recourse to measurement of other quantities functionally related to the measurand, the estimation of uncertainty is straight forward; as indicated in 15.1 above and summarised in figure 16.1. The latter covers the procedure completely, from the identification of the elementary errors of the measurement process, through to characterisation, summary and calculation of uncertainty.

16.4 Uncertainty Calculations for Indirect Methods of Measurement

Where the value of measurand is obtained from the measurement of other quantities, functionally related to the measurand, the method is considered to be indirect. Thus a quantity, G, may be determined as a function of other quantities, G1, .. Gn, through a functional relationship,

$$G = f(G_1, \dots \quad G_n)$$

where f ( ) represents the function concerned.

An example of an indirect method of measurement is the measurement of pressure, P, based upon separate measurements of force, F, and surface area, A, according to the functional relationship P = F/A.

Under such circumstances particular considerations have to be applied in calculating the uncertainty of measurement. First and foremost is the need to establish whether the components used in estimating the value of the measurand are functionally independent of one another, or otherwise dependent. It is also necessary to establish if the errors involved are statistically independent, or not, of the quantities concerned. Having established these facts it is further necessary to decide if we want to obtain the maximum uncertainty or the most likely uncertainty for the combination. The former is dealt with in the annex to this module. To estimate the most likely uncertainty values it is necessary to determine standard deviation values and combine them in accordance with the rules applied in the analysis of variance.

5.4.1 Functionally independent quantities

To be functionally Independent, the quantities within a given expression should not exhibit errors that are related one to the other.

If all of the quantities of an expression, $G = f(G_1, \dots G_n)$ are found to be functionally independent each component of error for $G_1 \dots \quad G_n$, can be treated independently, in the same manner as a direct measurement. Thus, the standard deviation values may be obtained for each component and then combined using the root-sum-square rule. Should all the errors for an indirect measurement, expressed in the form of standard deviation estimators, be statistically independent of the components, G_1, ... G_n, the variance expression can be readily determined from the function concerned.

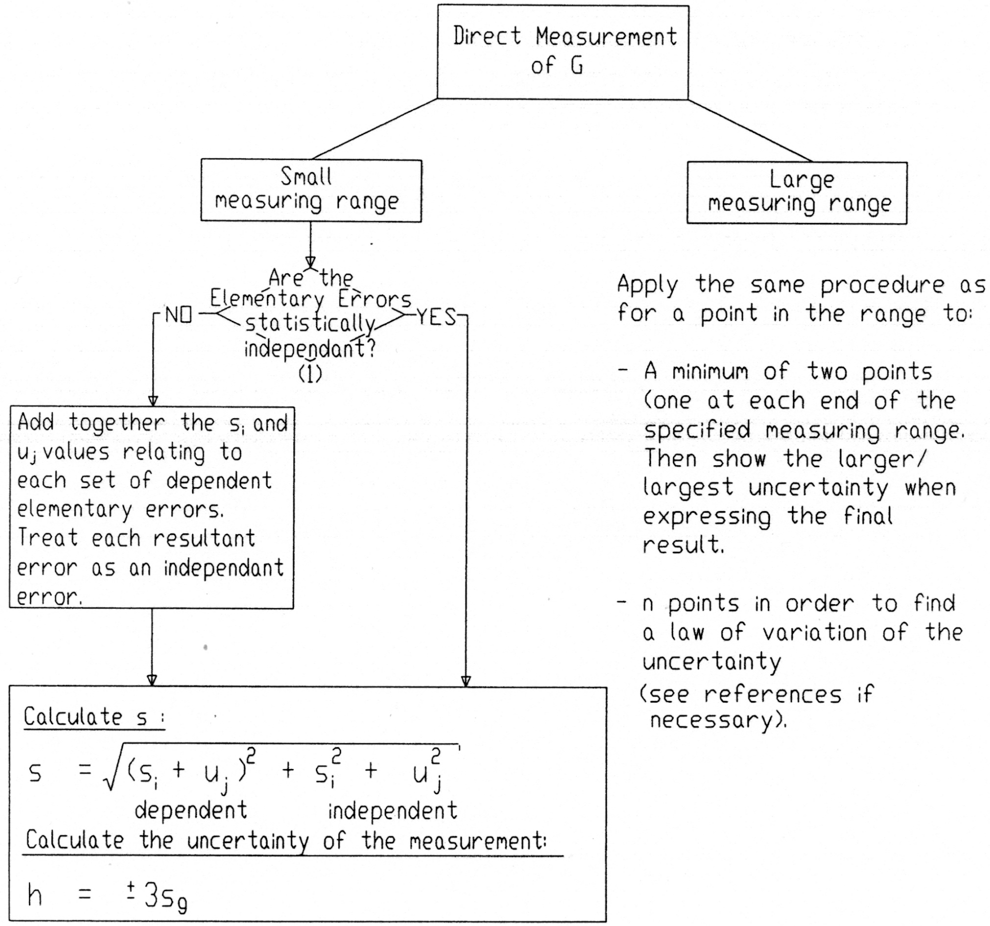
The most common functions encountered are sum and difference expressions, products and quotients. Consider, for example, the expression

$$G = aG_1 + bG_2 - cG_3$$

Here we have a sum and difference expression in which a, b and c are constants and G_1, G_2 and G_3 are variables. Standard deviation values, s_1, s_2 and s_3, may be determined for each of the variables and combined in accordance with the expression

$$s_g = \sqrt{(as_1)^2 + (bs_2)^2 + (cs_3)^2}$$

CALCULATION OF THE UNCERTAINTY OF MEASUREMENT FOR THE DIRECT MEASUREMENT METHOD.



(1) Each variable is influenced by a different cause.

For product of the form

$$G = G_1 \times G_2$$

Each component will exhibit an uncertainty expressed as 3 x standard deviation. Consequently, G will have an uncertainty, h_g, determined by h_1 and h_2 through the function:

$$(G + h_g) = (G_1 + h_1)(G_2 + h_2)$$

$$= G_1G_2 + G_1h_2 + G_2h_1 + h_1h_2$$

The component h_1h_2 may be considered to be small in comparison with G_1h_2 and G_2h_1 and so neglected.

The maximum value of h_g is $G_1h_2 + G_2h_1$ while the most likely value for h_g is $G_1h_2 - G_2h_1$.

Activity *Derive the most likely values for s_g for the following functions*

$$G = G_1/G_2$$
$$G = (G_1 \times G_2)/G_3$$

In the circumstances where errors are not found to be statistically independent it is necessary to separate dependent and independent errors, sum the dependent errors arithmetically and treat the sum as an independent component. Thus, in general terms,

$$s_g = (\sum A_d s_d)^2 + \sum (B_i s_i)^2$$

where $(\sum A_d s_d)^2$ is the square of the sum of the dependent components and $\sum (B_i s_i)^2$ is the sum of the squares of the independent components

Activity *Given that $G = aG_1 + bG_2 - cG_3$ and s_1 and s_2 are statistically dependent determine the expression for s_g.*

A summary of the procedures for dealing with functionally independent quantities is presented in figures 16.2 and 16.3.

16.4.2 Functionally dependent quantities

For functionally dependent quantities the analysis is somewhat more complicated and requires the identification of the dependent relationships. So an expression of the form,

$$G = f(G_1, G_2 \ldots G_n)$$

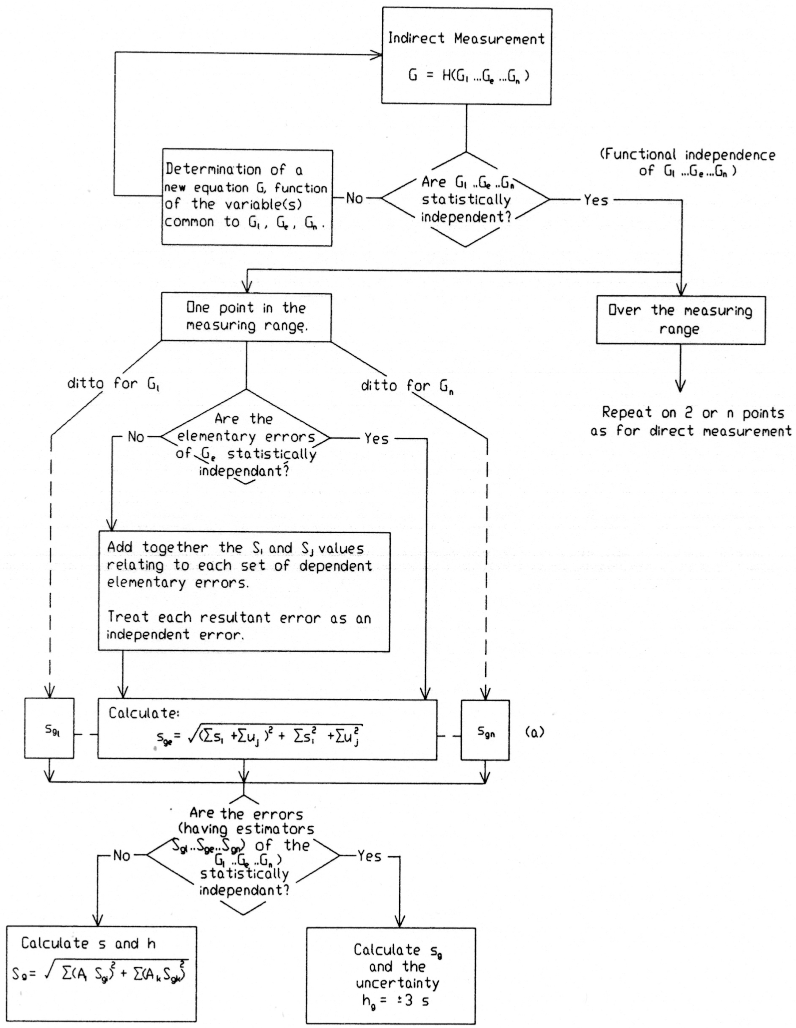
would be functionally dependent if relationships exist for each component in respect of common variables,

$$G_1 = f(x, y, z, t)$$
$$G_2 = f(x, y, z, t)$$
$$G_n = f(x, y, z, t)$$

Temperature is often encountered as a variable that renders functional quantities dependent.

Despite the difficulties that may be encountered in handling functionally dependent quantities in an indirect measurement a systematic approach to dealing with them can considerably ease the task.

CALCULATION OF UNCERTAINTY OF MEASUREMENT FOR THE INDIRECT MEASUREMENT METHOD



CALCULATION OF UNCERTAINTY: INDIRECT MEASUREMENT METHOD
SUMMARY OF PROCEDURE FOR ONE POINT IN THE MEASURING RANGE